

Are you a good fit for the *Icosahedron* group?

The *Icosahedron* group will be a challenging, fun experience for hard-core math lovers who already have a good understanding of basic high school math and who have a real drive to learn more, especially topics that are far from the standard curriculum. *It's an enrichment program, not an acceleration program.* To get the most out of this group, you need to have studied, and excelled in, geometry and advanced algebra (more advanced topics are good, too). Additionally, look at the following four questions. If at least two are challenging-but-fun-and-doable, you should be able to enjoy the program.

1 Undoubtedly you learned that the angle inscribed in a circle is equal to half the measure of the arc it subtends. Prove this fact, without looking it up!

2 Draw the graphs of the following on the Cartesian plane (the xy -plane).

(a) $|x - y| = 2$

(b) $|x| + |y| = 2$

(c) $|x + y| = 2$

(d) $\lfloor x \rfloor + \lfloor y \rfloor = 2$, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x ; for example, $\lfloor 9 \rfloor = 9$, $\lfloor \pi \rfloor = 3$, and $\lfloor -\pi \rfloor = -4$.

3 Find the number of zeros at the end of

$$1000! = 1 \cdot 2 \cdot 3 \cdots 999 \cdot 1000,$$

and explain why your answer is correct. No calculator or computer allowed. By the way, the answer is not 200.

4 Here are the first few rows of *Pascal's Triangle*.

$$\begin{array}{cccccccc}
 & & & & & & & 1 \\
 & & & & & & 1 & & 1 \\
 & & & & & 1 & & 2 & & 1 \\
 & & & 1 & & 3 & & 3 & & 1 \\
 & & 1 & & 4 & & 6 & & 4 & & 1 \\
 1 & & 5 & & 10 & & 10 & & 5 & & 1,
 \end{array}$$

where the elements of each row are the sums of pairs of adjacent elements of the prior row. For example, $10 = 4 + 6$. The next row in the triangle will be

$$1, 6, 15, 20, 15, 6, 1.$$

If you consider the first 1024 rows (i.e., all the way up to the row that begins $1, 1023, \dots$)

(a) How many terms will there be?

(b) How many terms will be odd?

Again, no calculator or computer allowed! Explain your answer.